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Missile Sizing for Ascent-Phase Intercept

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A computer code has been developed to determine the size of a ground-launched, multistage missile that can intercept a theater ballistic missile before it leaves the atmosphere. Typical final conditions for the interceptor are 450-km range, 60-km altitude, and 80-s flight time. Given the payload mass, which includes a kinetic kill vehicle, and achievable values for the stage mass fractions, the stage specific impulses, and the vehicle density, the launch mass is minimized with respect to the stage payload mass ratios, the stage burn times, and the missile angle-of-attack history subject to limits on the angle of attack, the dynamic pressure, and the maneuver load. For a conical body, the minimum launch mass is approximately 1900 kg. The missile has three stages, and the payload coasts for 57 s. A trade study is performed by varying the flight time, the range, and the dynamic pressure limits. A more detailed design is carried out for a particular missile to determine the heat-shield mass. The added heat-shield mass reduces the prescribed range by 100 km. Air-launching the same missile increases its range by 100 km. Finally, sizing the interceptor for air launch reduces its mass to approximately 1000 kg.

Nomenclature

a = speed of sound, m/s^2
C_A = axial-force coefficient
C_D = drag coefficient
C_L = lift coefficient
C_N = normal-force coefficient
D = drag, N
d = vehicle density, kg/m^3
g = acceleration of gravity, m/s^2
h = altitude, m
I_{sp} = specific impulse, s
L = lift, N
M = Mach number
m = mass, kg
q = dynamic pressure, kg/m^2
r = radial distance, m
S = base area, m^2
T = thrust, N
t = time, s
V = velocity, m/s
x = horizontal distance, m
α = angle of attack, deg
β = mass flow rate, kg/s^2
γ = flight-path angle, deg
δ = half cone angle, deg
ε = mass fraction
η = scale height, m
ρ = atmospheric density, kg/m^3
σ = stage payload mass ratio

Subscripts

B = burn
C = coast
F = fuel
f = final
P = payload
S = structure
s = sea level

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Introduction

A SERIOUS threat in today's world comes from short-range ballistic missiles. It is desirable to contemplate the design of an intercept missile that can destroy the ballistic missile as early as possible in the ascent phase. The primary question to be answered concerns the size of such an interceptor. How big does this interceptor have to be?

The approach followed here is similar to that of Ref. 1. After a missile shape is selected and achievable values are prescribed for the stage mass fractions, the stage specific impulses, and the vehicle mass density, the launch mass is minimized with respect to the stage payload mass ratios, the stage burn times, and the angle-of-attack history. Inequality constraints are imposed on the angle of attack, the dynamic pressure, and the maneuver load. The heat-shield mass is determined a posteriori, and its effect is included by reducing the standoff range. Although ground-launched missiles are the primary concern of this paper, air-launched missiles are discussed briefly.

In this paper, conceptual design considerations are discussed, and a physical model for the interceptor is presented. After the performance index and constraints are discussed, the minimum-launch-mass problem is stated, and numerical results are presented. Finally, a detailed design is carried out for a particular missile to determine the heat-shield mass and the effect of this mass on the range.

Conceptual Design

It is desired to size a missile that can intercept a short-range (400–600-km) ballistic missile in its ascent phase before it reaches exoatmospheric altitudes. For simplicity, the interceptor and the ballistic missile are assumed to operate in the same vertical plane. The intercept altitude is assumed to be 60 km; current theater ballistic missiles reach that altitude in 85–100 s. Allowing 20 s for launch detection, trajectory estimation, and downloading guidance information leaves 65–80 s of flight time for the interceptor. Based on the range of the target missile, a standoff range on the order of 450 km is desired for the safety of the launch platform. Typical values for the final distance, altitude, and time of the interceptor are taken to be 450 km, 60 km, and 80 s, respectively.

Because of its high average flight speed (5 km/s), the shape of the interceptor is chosen to be a cone of half angle 4.5 deg. This shape gives good structural rigidity, aerodynamic stability, and low drag. A cylindrical shape would have much less structural strength and would require fins for aerodynamic stability. Next, the missile is assumed to have four stages. If the mission requires fewer stages, the sizing code reduces the number of stages. Finally, the payload of the interceptor contains a kinetic kill vehicle and is assumed to have a total mass of 35 kg. To give the kinetic kill vehicle time to achieve the intercept, the payload is allowed to coast for 5 s before hitting the target.

Physical Model

Missile sizing is essentially a mass and aerodynamics problem in that statistical formulas are used to eliminate any need to consider the structure and the engines at this stage of the design process. Hence, it is assumed that the mass fraction of each stage, the specific impulse of each stage, and the vehicle density are known. Ranges of achievable values can be obtained from existing missiles.

Recall that the launch mass is being minimized with respect to the stage payload mass ratios, the stage burn times, and the angle-of-attack history. Hence, at the beginning of an optimization iteration, values of these quantities are either guessed or previously computed.

The equations of motion used to calculate the trajectory of the missile are those for flight in a vertical plane over a nonrotating spherical earth, that is,

$$\begin{aligned}\dot{x} &= \frac{r_s V \cos\gamma}{r_s + h}\\ \dot{h} &= V \sin\gamma\\ \dot{V} &= \frac{T\cos\alpha - D - mg\sin\gamma}{m}\\ \dot{\gamma} &= \frac{T\sin\alpha + L - mg\cos\gamma}{mV} + \frac{V\cos\gamma}{r_s + h}\end{aligned} \tag{1}$$

In this paper, thrust, drag, lift, and mass satisfy the functional relations $T = \text{const}$, $D = D(h, V, \alpha)$, $L = L(h, V, \alpha)$, and $m = m(t)$, so that the angle of attack is the only control.

For a spherical earth,

$$g = g_s\left(\frac{r_s}{r_s + h}\right)^2 \tag{2}$$

where $g_s = 9.81\ \text{m/s}^2$ and $r_s = 6{,}378{,}137$ m. The atmosphere is assumed to be exponential, so that the density satisfies the relation

$$\rho = \rho_s e^{-h/\eta} \tag{3}$$

where $\rho_s = 1.2255\ \text{kg/m}^3$ and $\eta = 7254$ m. The speed of sound is assumed constant at $a = 304.8$ m/s.

Given the payload mass (35 kg) and the payload mass ratio σ of each stage, the payload mass of each stage is given by

$$\begin{aligned}m_{P,4} &= m_{\text{payload}}\\ m_{P,k-1} &= m_{P,k}/\sigma_k, \qquad k = 4, \ldots, 1\end{aligned} \tag{4}$$

where $m_{P,0}$ is the launch mass. Next, the fuel mass and the structural mass of each stage are obtained from the relations

$$\begin{aligned}m_{F,k} &= \varepsilon_k(m_{P,k-1} - m_{P,k})\\ m_{S,k} &= \frac{1-\varepsilon_k}{\varepsilon_k} m_{F,k}\end{aligned} \tag{5}$$

where the mass fraction of each stage,

$$\varepsilon_k \triangleq \frac{m_{F,k}}{m_{S,k} + m_{F,k}} \tag{6}$$

is known.

Given the burn time of each stage, the constant mass flow rate is given by

$$\beta_k = m_{F,k}/t_{B,k} \tag{7}$$

so that, from the definition of specific impulse, the constant thrust of each stage becomes

$$T_k = g_s I_{\text{sp},k}\beta_k \tag{8}$$

where the specific impulse of each stage is known. A lower limit is imposed on the burn time to limit the propellant mass flow rate and the missile acceleration.

At this point, the mass during each stage is given by

$$m = m_{0,k} - \beta_k(t - t_{0,k}), \qquad t_{0,k} \le t \le t_{f,k} \tag{9}$$

where the initial mass of each stage is the final mass of the previous stage. Also, the initial time of each stage is given by

$$t_{0,1} = 0, \qquad t_{0,k} = \sum_{i=1}^{k-1} t_{B,i}, \quad k = 2, 3, 4 \tag{10}$$

and the final time of each stage is

$$t_{f,k} = t_{0,k} + t_{B,k} \tag{11}$$

Since the payload coasts for $t_C = 5$ s before hitting the target, the final time of the engagement is

$$t_f = t_{f,4} + t_C \tag{12}$$

The angle-of-attack history is represented by a piecewise linear function of time with three equally spaced nodes in each stage. Theoretically, the angle-of-attack history can have jumps at the stage times, but it is not practical to have a rapidly changing angle of attack near the stage points. Hence, the angle of attack is assumed to be continuous, meaning that the last node of one stage is the first node of the next. A total of nine nodes is needed for four stages.

The drag and the lift are computed from the relations

$$D = \tfrac{1}{2}C_D\rho S V^2, \qquad L = \tfrac{1}{2}C_L\rho S V^2 \tag{13}$$

In turn, the drag and lift coefficients are obtained from the axial- and normal-force coefficients as follows:

$$\begin{aligned}C_D &= C_A\cos\alpha + C_N\sin\alpha\\ C_L &= -C_A\sin\alpha + C_N\cos\alpha\end{aligned} \tag{14}$$

Initially, C_A was modeled as a constant, and C_N, as a function of α only (see Ref. 1). This model indicates that the optimal missile only has three stages. As a result, the angle of attack of the fourth stage is prescribed to be zero because it is just additional payload coast; only seven nodes are now needed to represent the angle-of-attack history.

To improve the aerodynamics, the methodology of Ref. 2 has been used to generate the axial- and normal-force coefficients of a conical body with 4.5-deg half angle as functions of angle of attack, Mach number ($M = V/a$), and Reynolds number. The resulting functions are approximated by

$$\begin{aligned}C_A &= C_{A0} + 1.22\times 10^{-7}h + 7.0\times 10^{-4}\alpha\\ C_N &= 0.035\alpha\end{aligned} \tag{15}$$

with h in meters and α in degrees. The quantity $C_{A0}(M)$ is obtained by linearly interpolating the following Mach-number table:

$$\begin{aligned}M &= 0.4, 0.9, 1.2, 2.0, 3.0, 5.0, 10, 15, 30\\ C_{A0} &= 0.16, 0.14, 0.26, 0.20, 0.145, 0.08, 0.04, 0.03, 0.025\end{aligned} \tag{16}$$

With this model, the drag and lift coefficients satisfy the functional relations $C_D = C_D(h, V, \alpha)$ and $C_L = C_L(h, V, \alpha)$.

Finally, the base area of each stage is obtained by setting vehicle density times stage volume equal to stage weight. The results are

$$S_k = \pi R_k^2, \qquad R_k = \left(\frac{3\tan\delta}{\pi}\frac{m_{P,k-1}g_s}{d}\right)^{\frac{1}{3}} \tag{17}$$

The initial conditions used for this study are the following:

$$t_0 = 0 \text{ s}, \quad x_0 = 0 \text{ m}, \quad h_0 = 0 \text{ m}$$
$$V_0 = 30.48 \text{ m/s}, \quad \gamma_0 = 85 \text{ deg} \tag{18}$$

where the initial velocity is due to a rail launch. The more accurate aerodynamic model consistently drives the launch angle toward 90 deg. Hence, the launch angle has been fixed at 85 deg to reduce the number of optimization parameters.

Finally, the achievable design values are chosen to be

$$\varepsilon_k = 0.85, \quad I_{sp,k} = 290 \text{ s}, \quad d = 9459 \text{ N/m}^3 \tag{19}$$

These values are optimistic but based on current technology; they have been chosen to reflect technology available in 10–15 years.

Performance Index and Constraints

The optimization problem is to minimize the launch mass, so that the performance index is given by

$$F = m_{P,0}/10{,}000 \tag{20}$$

where the scale factor makes F on the order of unity. Next, the prescribed final conditions are on the range (450 km), the intercept altitude (60 km), and the flight time (80 s). They are used in the scaled forms

$$C_1 = X_f/450{,}000 - 1 = 0$$
$$C_2 = h_f/60{,}000 - 1 = 0 \tag{21}$$
$$C_3 = t_f/80 - 1 = 0$$

where X_f and h_f are in meters.

In general, a stage payload mass ratio varies between 0 and 1. The value 0 means no payload, whereas the value 1 means no stage. To prevent the optimization code from generating values outside this range, inequality constraints are imposed on the stage payload ratios as follows:

$$C_{3+k} = \sigma_k - 0.1 \geq 0, \quad k = 1, \ldots, 4$$
$$C_{7+k} = 1.0 - \sigma_k \geq 0, \quad k = 1, \ldots, 4 \tag{22}$$

Next, each stage's burn time is required to be more than 2.0 s. This prevents a stage propellant mass flow rate from becoming infinite and limits the axial acceleration to reasonable values. Hence, additional inequality constraints are

$$C_{11+k} = t_{B,k}/2.0 - 1 \geq 0, \quad k = 1, \ldots, 4 \tag{23}$$

The angle of attack is limited to 10 deg. Since it is expected to be negative, the constraint is expressed as

$$C_{15+i} = 1 + \alpha_i/10 \geq 0, \quad i = 1, \ldots, 7 \tag{24}$$

with α_i in degrees. This constraint is imposed at each node.

Because of structural considerations, it is necessary to limit the dynamic pressure $q = \rho V^2/2$ and the maneuver load $q|\alpha|$, where α is in degrees. Path inequality constraints are converted to point constraints by computing the area of the constraint violation. For dynamic pressure, the constraint has the form

$$-\int_{t_0}^{t_f} \left[\max\left(\frac{q}{q_{\max}} - 1, 0\right)\right]^2 \mathrm{d}t \geq 0 \tag{25}$$

where the integrand is squared to make it look like a standard penalty function. A more logical form of the constraint is to require the constraint violation to be zero, that is, to require the integral to be zero. Unfortunately, that form does not always work in the optimization algorithm, because both the constraint and its derivatives are zero along a path with $q \leq q_{\max}$ everywhere. Equation (25) requires a negative number to be nonnegative. Hence, the optimization algorithm makes it as close to zero as the accuracy of the computation allows. Finally, the integral constraint is converted into a differential constraint, a boundary condition, and an algebraic constraint. If the variable P is defined as

$$\dot{P} = -100\left[\max\left(\frac{q}{q_{\max}} - 1, 0\right)\right]^2$$
$$P_0 = 0, \tag{26}$$

the constraint becomes

$$C_{23} = P_f \geq 0 \tag{27}$$

The scale factor 100 is used to make the constraint more visible to the optimization algorithm.

The maneuver load constraint is imposed in a similar manner. If the variable Q is defined by

$$\dot{Q} = -100\left[\max\left(\frac{q|\alpha|}{[q|\alpha|]_{\max}} - 1, 0\right)\right]^2$$
$$Q_0 = 0 \tag{28}$$

the constraint becomes

$$C_{24} = Q_f \geq 0 \tag{29}$$

Optimization Problem

The parameters used to minimize the launch mass are the stage payload mass ratios, the stage burn times, and the angle-of-attack nodes. These parameters are elements of the parameter vector

$$X = [\sigma_1 \cdots \sigma_4 \, t_{B,1} \cdots t_{B,4} \, \alpha_1 \cdots \alpha_7]^T \tag{30}$$

which contains a total of 15 parameters.

If values are guessed for the 15 unknown parameters, the differential equations, Eqs. (1), (26), and (28), can be integrated from the initial conditions, Eqs. (18), (26), and (28), to the final time, Eq. (12). This integration leads to values for the final states which make it possible to compute values for the performance index, Eq. (20), and the constraint functions, Eqs. (21–24), (27), and (29). Hence, the optimization problem or the nonlinear programming problem is to find the elements of the parameter vector X that minimize the performance index

$$J = F(X) \tag{31}$$

subject to the constraints

$$C_i(X) = 0, \quad i = 1, 2, 3$$
$$C_i(X) \geq 0, \quad i = 4, \ldots, 24 \tag{32}$$

The nonlinear programming code used in this study is based on recursive quadratic programming and is discussed in Ref. 3. The partial derivatives needed by the algorithm are computed by central differences.

Numerical Results

Converged results are presented in Table 1. The dynamic pressure and maneuver load limits imposed, that is, $q_{\max} = 2873$ kPa (60,000 lb/ft^2) and $(q|\alpha|)_{\max} = 9576$ kPa deg (200,000 lb/ft^2 deg), represent potential maximum values. For this missile, the payload mass ratios, the burn times, and the angle-of-attack nodes are given by

$$\sigma_k = 0.295, 0.260, 0.243, 1.00$$
$$t_{B,k} = 2.00, 5.04, 16.1, 51.8 \text{ s} \tag{33}$$
$$\alpha_i = -5.25, -3.00. -1.42, -3.71, -4.49, -8.71, -10.0 \text{ deg}$$

The minimum launch mass is 1880 kg. Note that the optimal missile has only three stages and that the actual coast time of the payload is 57 s. Also, the angle of attack reaches the $\alpha_{\max}$ limit at the end of the third stage, indicating that atmospheric effects are becoming

Table 1 Results of sizing computation

Stage	M_P, kg	M_F, kg	M_S, kg	M_O, kg	S, m^2	R, m	β, kg/s	Thrust, N
0	0.188E + 04							
1	0.554E + 03	0.113E + 04	0.199E + 03	0.188E + 04	0.875E + 00	0.528E + 00	0.563E + 03	0.160E + 07
2	0.144E + 03	0.349E + 03	0.616E + 02	0.554E + 03	0.388E + 00	0.351E + 00	0.693E + 02	0.197E + 06
3	0.350E + 02	0.924E + 02	0.163E + 02	0.144E + 03	0.158E + 00	0.224E + 00	0.573E + 01	0.163E + 05
4	0.350E + 02	0.000E + 00	0.000E + 00	0.350E + 02	0.615E − 01	0.140E + 00	0.000E + 00	0.000E + 00

t, s	x, km	h, km	V, km/s	r, deg	m, kg	Q	QA	α	C_D	C_L
Stage 1										
0.000	0.000	0.000	0.0305	85.0000	1879.7	0.6	3.0	−5.25	0.184	−.168
0.500	0.037	0.120	0.4830	70.4966	1598.0	140.5	579.8	−4.13	0.238	−.128
1.000	0.174	0.464	1.0111	66.7280	1316.4	587.4	1761.1	−3.00	0.139	−.098
1.501	0.451	1.063	1.6533	63.9300	1034.8	1446.0	3193.6	−2.21	0.079	−.074
2.001	0.923	1.971	2.4810	61.3251	753.2	2873.0	4075.7	−1.42	0.058	−.048
Stage 2										
2.001	0.923	1.971	2.4810	61.3251	554.4	2873.0	4075.7	−1.42	0.058	−.048
3.261	2.618	4.831	2.8132	57.0794	467.1	2490.4	6390.5	−2.57	0.055	−.087
4.521	4.848	7.912	3.2464	50.9028	379.8	2169.0	8054.3	−3.71	0.055	−.127
5.781	7.853	11.158	3.8122	43.5335	292.6	1912.0	7839.5	−4.10	0.057	−.140
7.041	11.916	14.496	4.6015	35.1778	205.3	1758.2	7889.3	−4.49	0.058	−.153
Stage 3										
7.041	11.916	14.496	4.6015	35.1778	143.7	1758.2	7889.3	−4.49	0.058	−.153
11.072	28.270	23.550	4.7600	23.7878	120.6	540.1	3562.5	−6.60	0.082	−.223
15.103	46.900	30.306	5.1476	16.3522	97.5	248.9	2166.4	−8.71	0.108	−.292
19.134	68.061	35.298	5.7347	10.3187	74.4	155.2	1451.6	−9.35	0.121	−.312
23.164	92.439	38.439	6.5924	4.2511	51.3	133.0	1330.3	−10.00	0.131	−.332
Stage 4										
23.164	92.439	38.439	6.5924	4.2511	35.0	133.0	0.	0.00	0.078	0.000
36.123	175.863	44.426	6.4205	3.8968	35.0	55.3	0.	0.00	0.086	0.000
49.082	257.671	49.763	6.3343	3.5082	35.0	25.8	0.	0.00	0.093	0.000
62.041	338.619	54.477	6.2862	3.1016	35.0	13.3	0.	0.00	0.100	0.000
75.000	419.058	58.580	6.2566	2.6848	35.0	7.5	0.	0.00	0.105	0.000
Payload										
75.000	419.058	58.580	6.2566	2.6848	35.0	7.5	0.	0.00	0.105	0.000
76.250	426.798	58.943	6.2544	2.6442	35.0	7.1	0.	0.00	0.106	0.000
77.500	434.535	59.301	6.2522	2.6036	35.0	6.7	0.	0.00	0.106	0.000
78.750	442.269	59.653	6.2502	2.5629	35.0	6.4	0.	0.00	0.106	0.000
80.000	450.000	60.000	6.2482	2.5221	35.0	6.1	0.	0.00	0.107	0.000

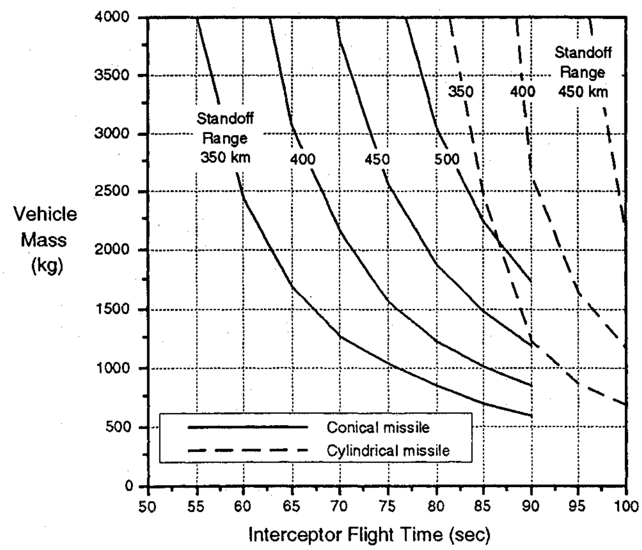

Fig. 1 Sizing results for a ground-launched interceptor missile.

small. Finally, the missile reaches the q limit at the end of the first stage but does not violate the $q|\alpha|$ limit because of the small angle of attack.

Figure 1 shows the results of a trade study for different values of the standoff range, the interceptor flight time, and the q and $q|\alpha|$ limits. The lower q limits represent those required for a cylindrical missile such as a sounding rocket and show that such a missile would have an excessive launch mass. The higher q limits are possible for conical missiles. These limits have been chosen to bracket the design space. The results show that the launch mass is highly dependent on the standoff range and the flight time, in other words, the average interceptor velocity. For a flight time of 80 s, the launch mass for the higher q limits varies between 650 kg for 350 km and 3100 kg for 500 km.

A more detailed design has been carried out for a three-stage missile with a 32-kg payload and a 1676-kPa (35,000-lb/ft^2) dynamic pressure limit. For this missile, the sizing code gives

$$\sigma_k = 0.320, 0.276, 0.272$$

$$t_{B,k} = 6.0, 5.2, 12.8 \text{ s} \tag{34}$$

$$\alpha_i = -2.60, -3.12, -3.60, -4.37, -5.25, -8.08, -10.0 \text{ deg}$$

and a launch mass of 1896 kg.

First, the missile trajectory is used to determine the amount of heat protection needed to keep the internal temperature less than about 400 K. Then, the mass fraction of each stage is obtained by starting with a motor mass fraction of 0.9 and adding the masses of the heat shield and the stage adapter to the structural mass. The resulting stage mass fractions and stage densities are given by

$$\varepsilon_k = 0.886, 0.827, 0.745$$

$$d_k = 8892, 9554, 11,890 \text{ N/m}^3 \tag{35}$$

Finally, the payload heat-shield mass and structural mass are subtracted from the payload mass, so that the mass available for the kinetic kill vehicle is 15 kg. A layout of the interceptor is shown in Fig. 2.

Second, the performance of the resulting missile is evaluated. Aerodynamic tables are generated, and atmospheric effects

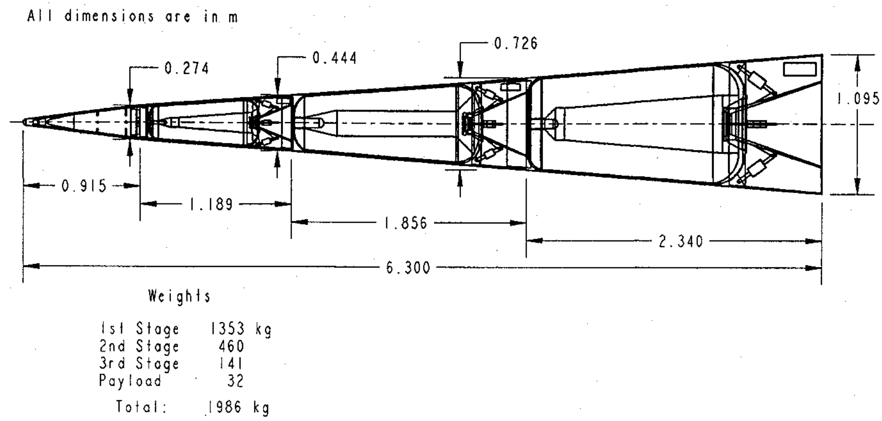

Fig. 2 Layout of ascent-phase interceptor concept.

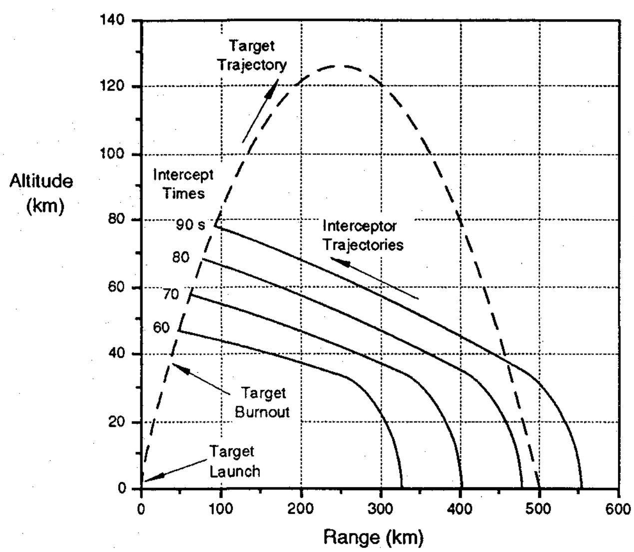

Fig. 3 Intercept trajectories for ascent-phase interceptor concept.

on engine performance are determined for engines with vacuum $I_{sp} = 290$ s. Then, the performance is obtained by computing the maximum final velocity trajectory for a given flight time to a given intercept altitude. The prescribed final time plus 20 s determines the position of the target at intercept and hence the final altitude of the interceptor. Trajectories have been computed for 60 through 90 s and have been placed on a trajectory plot of the target, a single-stage ballistic missile with a range of 500 km. As shown in Fig. 3, this missile can intercept the target within the atmosphere (60 km) from a standoff range of 400 km. Note, however, that the interceptor actually flies around 350 km, since the target is moving toward it. The reduced range relative to that predicted by the sizing code is due primarily to the added heat-shield mass and lower I_{sp} in the atmosphere.

Trajectories of the ground-launch design of Fig. 2 have been computed for the air-launch initial conditions $h_0 = 10{,}668$ m, $V_0 = 244$ m/s, and $\gamma_0 = 0.0$ deg. In general, the standoff range is increased approximately 100 km. Some of this range would be lost by having to make at least the first stage cylindrical for packaging the missile inside an aircraft. A sizing study has also been conducted for an air-launched interceptor. Even though the air-launched missile must withstand the same q limits as the ground-launched missile, the launch mass is reduced to approximately 1000 kg. For more details on missile sizing for air launch, see Ref. 4.

Conclusions

The purpose of this paper has been to determine the size of a ground-launched, multistage missile that can intercept a theater ballistic missile before it leaves the atmosphere. Typical final conditions for the interceptor are 450-km range, 60-km altitude, and 80-s flight time. Given the payload mass (35 kg), which includes a kinetic kill vehicle, and achievable values for the stage mass fractions (0.85), the stage specific impulses (290 s), and the vehicle density (9459 N/m^3), the launch mass is minimized with respect to the stage payload mass ratios, the stage burn times, and the missile angle-of-attack history subject to limits on the angle of attack (10 deg), the dynamic pressure (2873 kPa), and the maneuver load (9576 kPa deg). For a conical body, the minimum launch mass is approximately 1900 kg. The missile has three stages, and the payload coasts for 57 s. A trade study has been performed by varying the flight time, the range, and the dynamic pressure limits. It shows that all of these factors have a significant effect on the launch mass.

With the results of a sizing study for a 32-kg payload and $q_{max} =$ 1,676 kPa, a more detailed design using tabular aerodynamics and altitude-dependent thrust has been carried out to determine the heat-shield mass. The resulting missile has approximately 100 km less range than the sizing program predicted, primarily because of the additional mass required for heat protection and lower I_{sp} due to atmospheric effects. On the other hand, launching the same missile from an aircraft increases its range by approximately 100 km.

Sizing the interceptor for air launch with the same final conditions as the ground-launched missile reduces its launch mass to approximately 1000 kg.

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